INSTABILITY OF A SURFACE UPON EROSION IN A FLOW OF GAS WITH PARTICLES

A. P. Trunev and V. M. Fomin

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It is known that when plastic materials are eroded by flow of solid particles with a small angle of incidence, waves are formed on the specimen surface, the peaks of which are oriented perpendicular to the direction of particle motion [1-4]. There are various opinions on the nature of this phenomenon. It was proposed in [2] that waves on the material surface are caused by plastic deformation under the action of tangent stresses in the high-speed two-phase flow. It was demonstrated in [3] that the maximum erosion coefficient increases or decreases depending on the sign of the local surface curvature. It was concluded from this that the eroding surface is unstable relative to small perturbations of the erosion velocity, and that this is the cause of formation of finite amplitude waves. In [4] the relationship between wave formation and the behavior of the material during erosion (brittle or plastic loss of material) was established. It develops that some brittle materials become plastic at high loading rates, which leads to a change in the dependence of erosion rate on angle of incidence and development of microscopic ripples on the specimen surface. We will also note that the wave formation effect is also observed in the absence of a gaseous phase, i.e., microscopic waves are produced by the interaction of a flow of solid particles with a plastic obstacle [1].

In an analysis of the stability of the process of erosion of supersonic nozzles by a two-phase flow in [5] an equation was obtained to describe development of longwave perturbations in the system:

$$\partial y_w/\partial t = -D\left|\frac{\partial^2 y_w}{\partial x^2}\right|^q \frac{\partial^2 y_w}{\partial x^2}. \qquad (0.1)$$

This equation predicts that because of disruption of equilibrium in the two-phase flow above the curved system the given system will develop in an unstable manner. It will be shown below that in this case the characteristic wavelength of perturbations is of the order of the length of the dynamic particle relaxation zone ($\lambda \sim \ell_p$).

On the basis of the physical model of [3] we have formulated a mathematical model which describes microscopic waves on the surface of a plastic material. It develops that at small angles of incidence the model reduces to an equation of the form of Eq. (0.1). Moreover, if we reject the hypothesis of [3] in favor of that of [2], we also have an equation of the form of Eq. (0.1). Consequently, for erosion of a plastic material in a flow of gas with particles three different physical processes lead to a single result: the eroded surface proves to be unstable, and a perturbation-intensifying mechanism acts in the range of lengths of the order of the particle diameter ($\lambda \sim d_p$) and in the region $\lambda \sim \ell_p$, similar to the mechanism in a system with negative "viscosity."

1. Model of Wave Formation by Erosion. The model of [3] assumes that waves on the surface are produced solely by erosion, so that

$$\rho_* \partial y_w/\partial t = -\rho_p v_p E(\sin\alpha + \partial y_w/\partial x \cos\alpha). \qquad (1.1)$$

Here ρ_* is the density of eroding material; ρ_p, v_p are the density and the modulus of the velocity of the particle flow; E is the erosion coefficient; α is the angle of attack (Fig. 1). The model of [3] supposes removal of material due to plastic shear, on the basis of which the erosion coefficient was calculated as a function of angle of incidence and local radius of curvature of the surface, $E = E(\alpha_p, R)$. The fundamental parameter of the theory of [3] is the dimensionless complex $\lambda_F = m_p^{1/2} v_p/(2k_w^{1/2} R)$, where k_w is the effective rigidity of the

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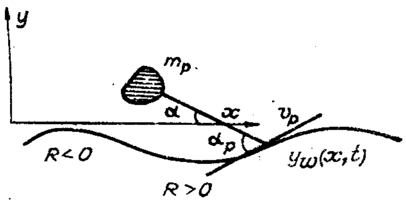

Fig. 1

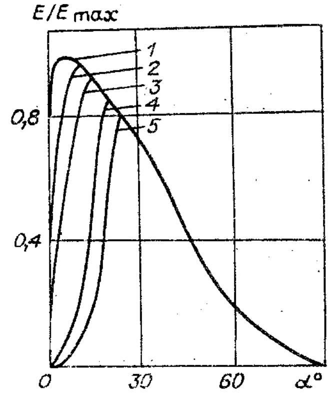

Fig. 2

surface and m is the particle mass. Figure 2 shows functions $E(\alpha_p)$ taken from [3], calculated for $\lambda_F = 0.2;\ 0.1;\ 0;\ -0.1;\ -0.2$ (curves 1-5). It is evident that in the range of small angles of incidence ($\alpha_p \approx 20°$) the erosion coefficient has a maximum, the value of which is higher, the greater the local radius of curvature of the surface. Since for low amplitude waves $1/R = \partial^2 y_w/\partial x^2$, in the first approximation we have

$$E(\alpha_p,\ R) = E_0(\alpha_p) + E_1(\alpha_p)\partial^2 y_w/\partial x^2, \tag{1.2}$$

where E_0, E_1 are nonnegative functions.

Substituting Eq. (1.2) in Eq. (1.1), we obtain

$$\partial y_w/\partial t = -G_0(\partial y_w/\partial x) - D_0(\partial y_w/\partial x)\partial^2 y_w/\partial x^2. \tag{1.3}$$

Here

$$G_0 = (\rho_p v_p E_0/\rho_*)\,(\sin\alpha + \partial y_w/\partial x \cos\alpha);$$
$$D_0 = (\rho_p v_p E_1/\rho_*)\,(\sin\alpha + \partial y_w/\partial x \cos\alpha).$$

(The angle of incidence is expressed in terms of the angle of attack and the local slope of the surface with the expression $\alpha_p = \operatorname{arctg}[(\operatorname{tg}\alpha + \partial y_w/\partial x)/(1 - \operatorname{tg}\alpha\partial y_w/\partial x)]$.) . Comparing Eq. (1.3) to Eq. (0.1), we may conclude that there is an analogy between the processes of wave formation in erosion in the range of scales $\lambda \sim d_p$ and $\lambda \sim \ell_p$. In other words, the mechanisms of perturbation intensification in the models of [3] and [5] are of the same type. This is explained foremost by the fact that the model of [3] is based on the equations of motion of a solid body in a flowing medium. In either case the reaction of the curvilinear surface leads to intensification or reduction in the erosion rate depending on the sign of the local curvature of the surface.

We note that Eq. (1.3) does not contain terms considering plastic deformation of the surface. This is an obvious shortcoming of the model of [3]. In reality erosion is a quite weak process occurring against the background of the intense process of deformation of the surface by solid particles. Below we will consider a model of wave formation with consideration of erosion and plastic deformations, obtained from the microscopic law of conservation of mass of the obstacle.

2. Microscopic Erosion Equation. Since the processes of erosion and surface deformation are caused by discrete particle-obstacle collisions, it is necessary to develop the conditions for a continuous description of these processes. Two characteristic time scales can be distinguished in this problem: $\tau_1 \sim m_p/(\rho_p v_p d_p^2)$, the interval between two successive collisions, $\tau_2 \sim \tau_1/E$, the characteristic erosion time. For erosion of metals $E \lesssim 10^{-3}$ for $v_p \lesssim 100$ m/sec, consequently $\tau_2 \gtrsim 10^3 \tau_1$ for the indicated collision conditions. In this case there exists an intermediate range of scales where the number of collisions is large, while erosion is still low, i.e., the range $\tau_1 \ll \tau \ll \tau_2$. We will assume that the true plastic flow satisfies the continuity condition

$$\partial\rho/\partial t + \partial\rho u/\partial x + \partial\rho v/\partial y + \partial\rho w/\partial z = 0. \tag{2.1}$$

Integrating Eq. (2.1) over the specimen thickness, we obtain

$$\frac{\partial}{\partial t}\int^{\tilde h}\rho dy + \frac{\partial}{\partial x}\int^{\tilde h}\rho u dy + \frac{\partial}{\partial z}\int^{\tilde h}\rho w dy = \tilde G, \quad (2.2)$$

where $\tilde h(x, z, t)$ is the instantaneous surface relief, $\tilde G$ is the true erosion rate.

Under erosion conditions the surface microrelief is changed in a random manner, since every particle-obstacle collision act is a random event. To eliminate the random component of the function $\tilde h(x, z, t)$, which changes at a frequency $\sim 1/\tau_1$, in Eq. (2.2) we average over time from t to t + τ and over an area of the surface with size $\Delta x \Delta z = \pi d_p^2/4$. Since $\tau \gg \tau_1$, the number of particles incident in the given area over the time τ will be quite large, and therefore the results of averaging will correlate with the parameters of the particle flow. If we assume that the density of the material changes only insignificantly, then after averaging, Eq. (2.2) takes on the form

$$\rho_* \partial y_w/\partial t + \partial J_x/\partial x + \partial J_z/\partial z = -\rho_p v_p E(\sin\alpha + \partial y_w/\partial x \cos\alpha). \quad (2.3)$$

Here J_x, J_z are the components of the mass flow vector due to plastic deformations. Equation (2.3) differs from Eq. (1.1) in the presence of divergent terms, describing the change in the surface due to plastic deformation of the obstacle. These terms vanish upon averaging over a specimen surface with sufficiently great area ($\Delta S \gg \pi d_p^2/4$). In this case we transform to the scale range $\lambda \sim \ell_p$, where Eq. (1.1) is also satisfied, this being one of the equations of the asymptotic model of [5], describing erosion in a two-phase flow.

To complete the model of Eq. (2.3) we assume that the particle flow is sufficiently rarefied, and therefore the effects of multiple collisions may be neglected. Then the expressions

$$J_x = Q(\alpha_p)n_p v_p(\sin\alpha + \partial y_w/\partial x\cos\alpha),\ J_z = 0, \quad (2.4)$$

are valid, where $Q(\alpha_p) = \int \rho' u' d^3x dt$ describes the shift in mass along the surface upon collision of an individual particle with the obstacle. Combining Eqs. (2.3), (2.4), we obtain

$$\begin{gathered}\partial y_w/\partial t = -G(\partial y_w/\partial x) - D_1(\partial y_w/\partial x)\partial^2 y_w/\partial x^2, \\ D_1 = (n_p v_p/\rho_*)\left[Q\cos\alpha + (\sin\alpha + \partial y_w/\partial x\cos\alpha)\, dQ/d\alpha_p\right].\end{gathered} \quad (2.5)$$

It is clear from general considerations that $Q(0) = Q(\pi/2) = 0$. Consequently, this function has a maximum at some $\alpha_p = \alpha^*$. But then in the interval $0 < \alpha < \alpha^*$ the condition $dQ/d\alpha_p > 0$ is satisfied, so that $D_1 > 0$. Thus, here we have the phenomenon of negative "viscosity" again. Generally speaking, Eq. (2.5) is more physical than Eqs. (0.1) or (1.3), in which the coefficients of the highest derivatives are constant in sign. It can easily be shown that $D_1 < 0$ as $\alpha_1 \to \pi/2 - 0$. This means that Eq. (2.5) is a variable type equation [6]. Using the model of [3], one can calculate the functions G(R) and Q(R). Then in the general case Eq. (2.3) takes on the form

$$\partial y_w/\partial t = -G_0(\partial y_w/\partial x) - \tilde D\partial^2 y_w/\partial x^2 - k\partial^3 y_w/\partial x^3, \quad (2.6)$$

where $\tilde D = D_0 + D_1 - 3\,(n_p v_p/\rho_*)\ \partial y_w/\partial x\ (\sin\alpha + \partial y_w/\partial x\ \cos\alpha)\ dQ/dR;\ k = (n_p v_p/\rho_*)(\sin\alpha + \partial y_w/\partial x\cos\alpha)\, dQ/d(1/R)$.

It is evident from Fig. 2 that $D_1 \to 0$ as $\alpha_p \to \pi/2$. Therefore the function $\tilde D$ changes sign in the interval $0 < \alpha_p < \pi/2$. The properties of the solutions of Eq. (2.6) have been studied in detail by many authors (this is a Korteweg–de Vries type equation). It is known, in particular, that as $k \to 0$ the peaks of the traveling waves become sharper, changing into the system of [7]. It is curious that such an effect has been observed experimentally in erosion of a plane wedge in a dust flow [8].

<u>3. Mechanisms for Development of Surface Instability in a Two-Phase Flow</u>. We will now consider in greater detail the question of stability of a body surface during erosion in a flow of gas with particles. We will commence from the equations of the asymptotic model developed in [5]:

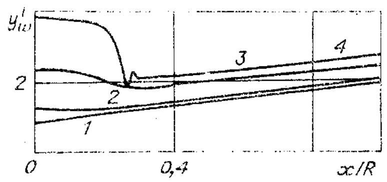

Fig. 3

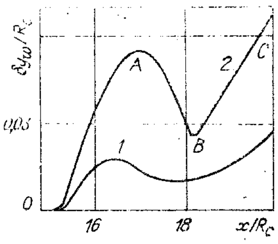

Fig. 4

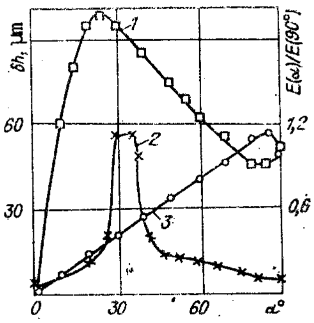

Fig. 5

$$l_s \frac{\partial^2 y_s}{\partial x^2} + \frac{\partial y_s}{\partial x} - \frac{\partial y_w}{\partial x} + (y_s - y_w)\frac{\partial}{\partial x}\ln(\rho u y^\nu)_w = 0,$$
$$\rho_* \frac{\partial y_w}{\partial t} = \sum_{s=1}^{N} \rho_s u_s E_s\left(\frac{\partial y_s}{\partial x} - \frac{\partial y_w}{\partial x}\right). \tag{3.1}$$

Here $y_w(x, t)$ is the boundary surface; $y_s(x, t)$ is the particle trajectory; ℓ_s is the dynamic relaxation parameter; $(\rho u y^\nu)_w$ is the gas flow in the flow region near the wall; N is the number of particle fractions; $\nu = 0$ and 1 in planar axisymmetric flows respectively.

The basic result of [5] concerns behavior of long wave perturbations in the model of Eq. (3.1). In long waves system (3.1) can be reduced to system (0.1) with all subsequent conclusions. In Eq. (3.1) we take $l_s=\text{const}$, $\nu = 0$, $(\rho u)_w=\text{const}$, $\rho_s u_s=\text{const}$, $E_s = (u_\infty^2/\sigma_e)\,|y'_s - y'_w|^q$, $y'_{s,w} \ll 1$, $s = p = 1$. With these assumptions Eq. (3.1) describes erosion of a planar thin profile in a monodispersed flow. In contrast to the model developed in [9], system (3.1) is valid not only at low, but also moderate particle flow rates. After some transformations we obtain from Eq. (3.1)

$$l_p \eta_{xt} + l_p^{q+1} G\,|\eta_x|^q \eta_{xx} + \eta_t = 0,$$
$$\partial y_w/\partial x = l_p \eta_x + \eta, \tag{3.2}$$

where $\eta = v_p/u_p$ is the slope of the particle trajectory; $G = \rho_{p\infty} u_\infty^3/(\rho_* \sigma_e)(q+1)$ is the characteristic erosion rate; $l_p = \rho_s^0 d_p^2 u_\infty/18\mu_\infty$; ρ_s is the density of the particle material; μ_∞ is the dynamic viscosity of the gas.

In Eq. (3.2) we take $\eta_x = \eta_x^0 + \tilde{\eta}_x$, $\eta_x^0 = \text{const}$ (parabolic profile), $\tilde{\eta}_x \ll \eta_x^0$, $\tilde{\eta} \sim \exp(ikx - i\omega t)$, and find the monochromatic perturbation spectrum of the corresponding linearized system:

$$\omega(k) = G\,|\eta_x^0|^q l_p^{q+1} k^2 (i + l_p k)/(1 + l_p^2 k^2). \tag{3.3}$$

Since Im $\omega > 0$, system (3.2) is unstable in the vicinity of the initial state $\eta_x(x, 0) = \eta_x^0$. The characteristic time for instability development $\tau^* = l_p/G(\eta_x^0 l_p)^q$. Hence, with the assumption $\eta_x^0 = 1/R$, where R is the local radius of curvature of the wall, we have an estimate of the time for instability development in erosion of a supersonic nozzle [5]. It is evident from Eq. (3.3) that the characteristic wavelength is of the order of magnitude of the length of the dynamic relaxation zone $\lambda \sim \ell_p$.

Analyzing the role of individual terms in the first expression of Eq. (3.2), we conclude that with interaction of the first and second terms formation of an intense discontinuity in the system is possible. The expressions on this discontinuity have the form

$$[\eta]=0,\quad [y_w']=l_p[\eta_x],\quad Gl_p^q[\eta_x^{q+1}]=c[\eta_x], \tag{3.4}$$

where $[\eta_x]=\eta_x^+-\eta_x$ is the amplitude of the discontinuity: c is the velocity or propagation of the discontinuity.

At q = 1 from Eq. (3.4) we find $c = G[y_w']$. From this it follows that the ledges formed upon erosion of nozzles by monodispersed particles [5] should shift in the direction toward the nozzle mouth. In numerical calculations nonlinear effects appear before the linear mechanism of perturbation intensification can develop. Figure 3 shows the results of numerical calculations of erosion of a parabolic profile within the framework of the model of Eq. (3.2). Curves 1-4 show the change in slope of the profile at times 0, 0.2, 0.6, 0.8 τ^*. It is evident a strong discontinuity in the derivative of the profile (ledge) is formed, as a consequence of which it is difficult to distinguish linear effects occurring at $t \geq \tau^*$. Figure 4 presents the results of numerical calculations of the mass removed $\delta y_w = y_w(x, t) - y_w(x, 0)$ for erosion of a supersonic nozzle with a parabolic directrix in the final portion, as performed with the model of Eq. (3.1). In this example N = 100, and the calculation was performed by an explicit finite difference method of first order accuracy. Curves 1, 2 were calculated for t = 1.42 and 3.5 sec. Also shown is a singularity in the behavior of initially smooth perturbations in models of the type of Eq. (0.1). One can clearly see the sharpening of the wave structure, from which crests are formed – the segment ABC in curve 2.

Summarizing the above, we can make two important conclusions regarding the properties of systems such as Eq. (3.1): due to disruption of dynamic symmetry in the two-phase flow above the curved surface, intensification of initial perturbations occurs, and because of the nonlinear properties of the transport operator, upset of the initially smooth perturbations occurs with formation of intense discontinuities in the derivative of the contour.

4. Effect of Fine Scale Perturbations on Macroparameters of the Process. As has already been noted above, transition to the scale region $\lambda \sim \ell_p$ is accomplished in Eq. (2.3) by averaging over an area $\Delta S \gg \pi d_p^2/4$. We will now pose the problem of determining the effective erosion parameters for an originally planar specimen with specified surface microrelief $y_w(x, z, t)$. If in the scale region $\lambda \sim \ell_p$ the specimen remains planar, then after averaging Eq. (2.3),

$$\rho_*\partial\langle y_w\rangle/\partial t=-\rho_p v_p(\langle E\rangle\sin\alpha+\langle E\partial y_w/\partial x\rangle\cos\alpha)=-\rho_p v_p E_{\text{eff}}\sin\alpha. \tag{4.1}$$

Thus the effective (observed in experiment) erosion coefficient is by definition

$$E_{\text{eff}}=\langle E\rangle+\langle E\partial y_w/\partial x\rangle \operatorname{ctg}\alpha. \tag{4.2}$$

We will assume that the dependence of the true erosion coefficient on angle of incidence for the given material has the form $E \sim (\sin \alpha_p)^q$, typical of brittle removal [4]. A similar dependence is observed in the case of high-speed interaction ($v_p \gtrsim 1$ km/sec) [9]. Averaging in Eq. (4.1) we obtain

$$\begin{gathered}E_{\text{eff}}=\sin\alpha\,\langle\cos_w\rangle+\cos^2\alpha\,\langle\sin^2\alpha_w/\cos\alpha_w\rangle/\sin\alpha \text{ at } q=1,\\ E_{\text{eff}}=\sin^2\alpha\,\langle\cos^2\alpha_w\rangle+3\cos^2\alpha\,\langle\sin^2\alpha_w\rangle \text{ at } q=2,\end{gathered} \tag{4.3}$$

where $\tan \alpha_w = \partial y_w/\partial x$. Commencing from Eq. (4.3), we can interpret the experimental results of [4] in the following manner. In the brittle destruction region surface roughness is negligible, the contribution of the second term on the right side of Eq. (4.3) is small, and therefore $E_{\text{eff}} \sim \langle(\sin \alpha)^q$. Upon transition to plastic behavior waves are formed on the specimen surface, whence the role of the second term increases abruptly and the dependence of the erosion coefficient on angle of incidence at $\alpha \ll 1$ under these conditions is determined by the parameters of fine scale perturbations, $E_{\text{eff}} \sim \langle\sin^2\alpha_w\rangle$.

Figure 5 shows experimental dependences of wave amplitude for ash erosion of heater tubes [2] (curve 2) and the erosion coefficient of glass [4] in a flow of SiC particles $v_p \simeq 152$ m/sec) for the case of brittle ($d_p \cong 216$ μm) and plastic ($d_p \cong 9$ μm) erosion (curves 3 and 1, respectively). There is a correlation between the curves $E_{\text{eff}}(\alpha)$ and $\delta h(\alpha)$ for plastic erosion (curve 1), where finite amplitude waves are formed on the surface [4]. On the other hand, the indicated relationship follows from the estimate $\langle \sin^2 \alpha_w \rangle \sim (\delta h/\lambda)^2$ and Eq. (4.3).

Thus it has been established that fine scale perturbations have a significant effect on the behavior of the effective erosion coefficient, especially in the region of small angles of incidence. These results are a basis for constructing a complete model of erosion of plastic materials, which must include a model of particle interaction with the obstacle, Eq. (2.6), and a procedure for averaging Eq. (4.2) for comparison of $E_{eff}(\alpha, v_p)$ with available experimental data.

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